BROWN-HET-1593

Punctuated eternal inflation via AdS/CFT

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Abstract

The work is an attempt to model a scenario of inflation in the framework of Anti de Sitter/Conformal Field theory (AdS/CFT) duality, a potentially complete nonperturbative description of quantum gravity via string theory. We look at bubble geometries with de Sitter interiors within an ambient Schwarzschild anti-de Sitter black hole spacetime and obtain a characterization for the states in the dual CFT on boundary of the asymptotic AdS which code the expanding dS bubble. These can then in turn be used to specify initial conditions for cosmology. Our scenario naturally interprets the entropy of de Sitter space as a (logarithm of) subspace of states of the black hole microstates. Consistency checks are performed and a number of implications regarding cosmology are discussed including how the key problems or paradoxes of conventional eternal inflation are overcome.

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I. INTRODUCTION

Making contact with realistic cosmology remains the fundamental challenge for any candidate unified theory of matter and gravity. Precise observations [1, 2] indicate that the current epoch of the acceleration of universe is driven by a very mild (in Planck units) negative pressure, positive energy density constituent - "dark energy". Likewise there is strong evidence that the Big Bang phase of the universe was preceded by a exponentially accelerated (inflation) phase [3, 4] driven by a negative pressure, positive energy fluid. In most cosmological models this inflationary stage is brought about by a scalar field coupled to gravity, slowly "rolling down" an almost flat potential hill, and upon reaching the bottom of the hill gives rise to the big-bang stage. In most such models inflation is inevitably "future eternal" i.e. there are always some residual regions which keep on rapidly inflating. The steady state picture that emerges is a fractal multiverse structure with many "pocket universes" causally separated by continually inflating sterile regions. These "pocket universes" might possess all possible different "fundamental constants" of nature (including a cosmological constant). So in this scenario one now talks about "environmental constants" of nature instead.

(Super)string theory is the currently leading candidate for an unified theory of fundamental interactions and should be compatible with realistic cosmology if it has to be anything more than an attractive toy model. It is great news that string theory appears to have an enormous "landscape" of vacua including many long-lived metastable states with positive cosmological constants [5] forming a quasi-continuum [6–9] which are highly relevant for realistic cosmology. These metastable phases ultimately decay to neighboring stable anti-de Sitter phases via "rolling down the hill" and quantum mechanical tunneling and this is exactly what one needs to attempt a fundamental physics model of the (inflationary) multiverse. So it is important to develop tools in the fundamental theory which describe such transitions.

The AdS/CFT formulation of String theory [10] is a potentially nonperturbative definition of superstring theory in asymptotically AdS spaces and is an perfect setting to embed the string theory landscape. The neighborhood of the landscape that we study is summarized in figure 1.

The first step within this approach was taken in [11] which looked at classical AdS-Reissner-Nordstrom/dS domain wall type geometries (an inflating dS bubble interior patched

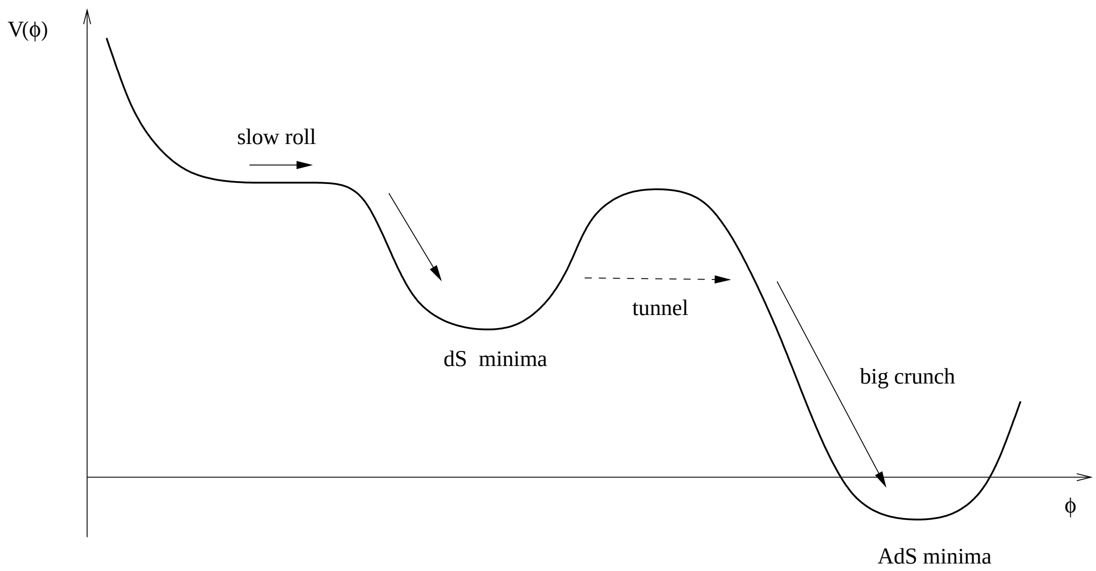

Figure 1: The potential landscape showing the dS metastable and AdS stable vacua. The arrows indicate the time evolution of a possible worldline.

to an asymptotic AdS black hole spacetime) in the thin wall approximation. Classical bubble spacetimes without charge were further investigated in [12] and subsequently in [13]. The latter has an added attractive feature that the mysterious Gibbons-Hawking entropy of de Sitter space could be identified as a (logarithm of) subspace of black hole microstates. The aim of the present paper is to build on the work of [13] by attempting to identify the CFT states which code these inflating states in a thermal ensemble, and further explore the consequences of this scenario.

The paper is organized as follows. In section 2, we review the semiclassical consistency condition of [13] which arises from interpreting de Sitter entropy within a unitary framework for quantum gravity. This amounts to the condition that the number of states available must be bounded below by the Gibbons-Hawking entropy if states corresponding to a truly semiclassical region of de Sitter space exist. . Then in section 3, we review the classical bubble spacetimes. Keeping in mind the dual CFT as the underlying quantum description we are forced to rule out the whole class of time-symmetric bubble solutions based on the consistency condition. In particular, this rules out tunneling solutions of the type described in [14]. In section 4, we focus on the CFT picture of the time-asymmetric solutions compatible with the aforementioned consistency condition and their signatures in the dual CFT. We model the effect of the radiation reflecting off the shell by reducing the problem to that

of a moving mirror in a black hole background. In section 5 we discuss consistency checks of such a scenario. In section 6 we draw various conclusions regarding the implications for such a picture for cosmology and how several problems/paradoxes that plague the eternal inflation scenario - namely the measure problem, the youngness paradox and the Boltzmann brains paradox are overcome in this setting.

II. QUANTUM DE SITTER STATE

In [13] it had been advocated that the Gibbons-Hawking entropy of dS space can be correctly interpreted as the logarithm of the dimension of quantum gravity state (Hilbert) space. This proposition was based on numerical observations of the entropy of the present universe, dominated by a contribution from the cosmic microwave background (CMB) photons. We review this rationale briefly here once again. The entropy of CMB photons see, for example, [15]) is,

$$S_{CMB} \sim 10^{88}$$

with wavelength, $\lambda_{CMB} \sim 1$ mm, while the Gibbons-Hawking entropy of dS space with the present day cosmological constant is,

$$S_{GH} \sim 10^{123}.$$

So with a field theory cut off with about $\epsilon = 1$ TeV, the total number of states in effective field theory associated with CMB photons is $(\lambda_{CMB}\epsilon)^3 S_{CMB} \sim 10^{138}$, way too large to be accounted by the de Sitter entropy.

From past experience with AdS/CFT we know that the notion of a cut off in the bulk effective field theory (EFT) is complex and in particular dependent on the respective quantum (CFT) state, unlike the conventional uniform cut off in usual effective field theories [16–19]. So an EFT in the bulk with an adaptive step size cutoff which allocates up to 10^{35} states per CMB photon would saturate the Gibbons-Hawking entropy of dS space with the present day cosmological constant. Hence we are justified in saying that Gibbons-Hawking de Sitter entropy does count (logarithm of) the number of possible states in EFT, albeit with a clever cut-off [13].

AdS/CFT also taught us that generic states in the quantum theory (CFT) have no nice bulk space time interpretation. But in addition to just a smooth space time geometry (metric) once also requires large families of observables to reproduce the entire span of semiclassical physics on this smooth geometry. This observation coupled with the above numerical bound on the entropy of the universe leads us to the following hypothesis for the de Sitter state in a quantum theory of gravity:

- **Semiclassical Consistency Hypothesis (SCH):** *The set of microstates representing a semiclassical de Sitter region must number at least* $e^{S_{GH}}$.

III. INFLATION IN AdS/CFT: dS/SAdS DOMAIN WALLS

Let us briefly review the geometric set up which can be found in great detail in [11, 12, 20]. We have a domain wall geometry by patching together a (interior) dS-region with cosmological constant (c.c.) $\Lambda_{dS} = \frac{3}{l_{dS}^2}$ and an (exterior) AdS-Schwarzschild geometry with c.c set to $\Lambda_{AdS} = \frac{3}{l_{AdS}^2}$ and hole mass M across a thin spherical shell (parametrized $r = R(\tau)$ where τ is the proper time of the shell. The world volume metric of the shell is then given by the equation,

$$ds^2 = -d\tau^2 + R(\tau^2)d\Omega^2 \,.$$

The geometry is specified by the following metric and the coordinate ranges,

$$ds^2 = \begin{cases} -f_{dS}(r)dt_{dS}^2 + \frac{dr^2}{f_{dS}(r)} + r^2 d\Omega^2, & f_{dS}(r) = 1 - \frac{r^2}{l_{dS}^2}, \qquad r < R(\tau) \text{ (inside)} \\ -f_{SAdS}(r)dt_{SAdS}^2 + \frac{dr^2}{f_{SAdS}(r)} + r^2 d\Omega^2, & f_{SAdS}(r) = 1 - \frac{2GM}{r} + \frac{r^2}{l_{AdS}^2},\ r > R(\tau) \text{ (outside)} \,. \end{cases}$$

Here r is a global coordinate and the times inside and outside the bubble are related by equating the invariant distance along the shell from inside and outside.

$$\left(-f_{dS}(r)dt_{dS}^2 + \frac{dr^2}{f_{dS}(r)}\right)_{r=R(\tau)} = -d\tau^2 = \left(-f_{SAdS}(r)dt_{SAdS}^2 + \frac{dr^2}{f_{SAdS}(r)}\right)_{r=R(\tau)} . \qquad (1)$$

The consistent junction conditions relate the discontinuity of the extrinsic curvature across the shell to the stress energy tensor of the shell (defining $\kappa = 4\pi G\sigma$, with σ the surface energy density of the shell),

$$\sqrt{\dot{R}^2 + f_{SAdS}(r)} - \sqrt{\dot{R}^2 + f_{dS}(r)} = \kappa R$$

which determine the shell trajectory (substituting $r = R(\tau)$) can be rewritten

$$\dot{r}^2 + V_{eff}(r) = 0 \tag{2}$$

so the trajectory is that of a zero net energy particle moving in potential,

$$V_{eff}(r) = -\frac{\left(\frac{1}{l_{dS}^2} + \kappa^2 - \frac{1}{l_{AdS}^2}\right)^2 + \frac{4}{l_{dS}^2 l_{AdS}^2}}{4\kappa^2} r^2 + 1 + \frac{GM\left(\frac{1}{l_{dS}^2} + \frac{1}{l_{AdS}^2} - \kappa^2\right)}{\kappa^2}\frac{1}{r} - \frac{G^2M^2}{\kappa^2 r^4} \tag{3}$$

in one space dimension. The solutions can be broadly classified into two categories - one when the parameters are such that the potential has a maximum that is positive and when it is negative. In the first case the zero energy particle is reflected off the barrier and the bubble traces out a time symmetric trajectory, in some choice of coordinates, while in the second case the particle rolls over to the top of the potential hill to the other side and the bubble traces out a time asymmetric trajectory , figure 2.

As shown in [12] for all the time symmetric cases:

$$r_{bh} < r_{dS}$$

i.e. the black hole horizon radius is less than the de Sitter horizon length and this implies,

$$S_{GH} > S_{BH} \,,$$

with S_{BH} the Bekenstein-Hawking entropy of the black hole.

Now let us consider a black hole such that $r_{bh} > l_{AdS}$ so that the black hole is well-described by the canonical ensemble in the CFT. The available microstates, $e^{S_{BH}}$, must exceed the microstates of the internal dS bubble, $e^{S_{GH}}$, according to the SCH. There are simply not enough microstates in the dual CFT to constitute the whole range of semiclassical phenomena in the interior of the bulk dS bubble. So the SCH rules out all such time symmetric bubble configurations in the quantum picture. In particular no such false vacuum dS bubbles can be formed by quantum mechanical tunneling from AdS spacetime. Thus we see in the AdS/CFT framework, the type of tunneling events considered in [14, 21] do not occur. Once any kind of local tunneling process allows arbitrarily large regions of

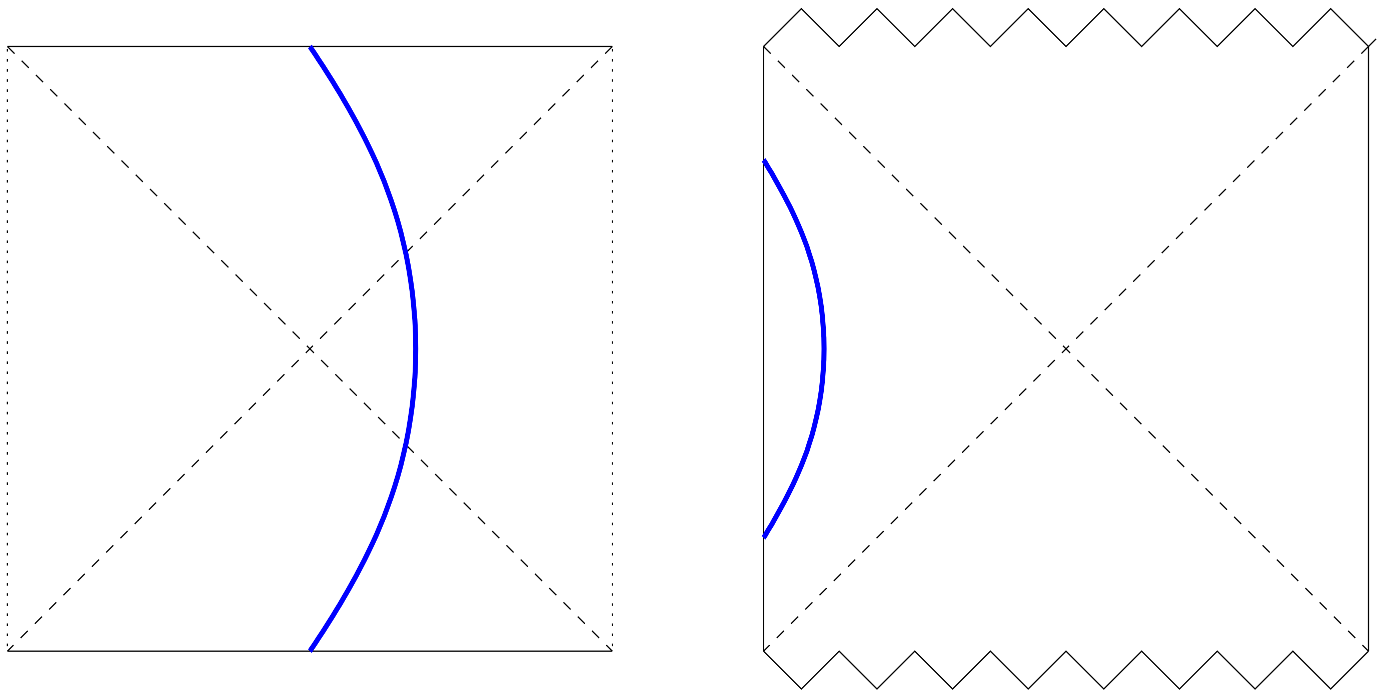

Figure 2: A Time symmetric bubble trajectory: The left diagram is dS spacetime and the right diagram is SAdS spacetime. The blue curve is the bubble trajectory. The domain wall geometry is made up of the region to the left side of the blue curve in the dS spacetime diagram and to the right side of the blue curve of the AdS spacetime diagram.

semiclassical spacetime to be formed, we quickly violate unitarity, so this is an important new constraint that arises from the CFT viewpoint, that would not be seen via a straightforward semiclassical gravity analysis.

We are left with the time asymmetric situation where the bubble emerges from the past singularity $R(\tau = 0) = 0$ and keeps on expanding indefinitely i.e. $r(\tau \to \infty) \to \infty$ to simulate an eternally inflating bubble of de Sitter space inside an asymptotic AdS space. The conformal diagram of such a geometry is shown in figure 3.

This can be ensured if the outward normal points to increasing r when $r \to \infty$ i.e. by having $\beta_{dS} > 0$ which gives

$$\kappa^2 > \frac{1}{l_{dS}^2} + \frac{1}{l_{AdS}^2}\,.$$

For these time asymmetric cases Gibbons-Hawking dS entropy can be accounted for by the black hole microstates (i.e. one can arrange for $r_{bh} > l_{dS}$) by having

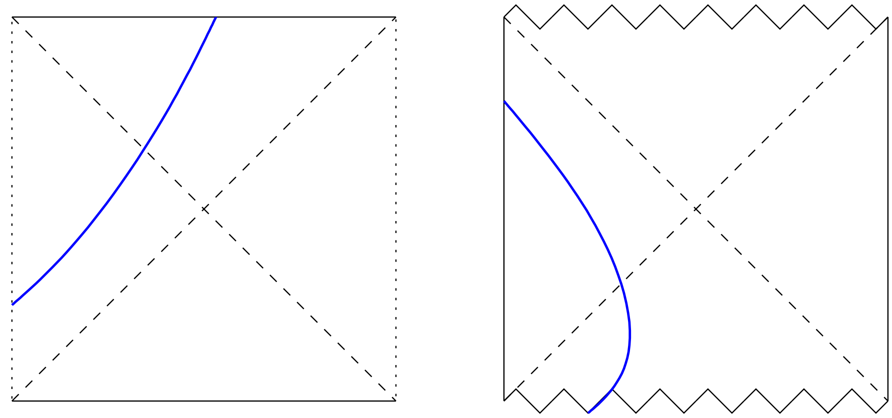

Figure 3: Time asymmetric trajectory : The interior dS region is shown on the left, and the exterior SAdS region is shown on the right. The domain wall geometry is made up of the regions to the left of the blue curve in the dS diagram and to the right of the SAdS diagram joined across the bubble wall (blue curve).

$$\left(2GMl_{AdS}^2\right)^{1/3} > l_{dS}.$$

After taking into account back reaction as was shown in [13], the left asymptotic boundary of the bubble exterior ceases to exist as this region is unstable to perturbations and a singularity develops. In the bubble interior, any worldline eventually tunnels back into the stable AdS vacuum, but the tunneling portion of the bubble undergoes a big crunch [22].

IV. THE CFT PICTURE

A. Schwarzschild anti-de Sitter black hole

The basic setup we consider is shown in figure 5. A large black hole is formed from the collapse of a null shell sent in from infinity into empty AdS_4 spacetime. This process is expected to be described by a pure state within a single conformal field theory. Under time evolution, the state is thermalized, and late-time correlation functions will be well-approximated by those in the canonical ensemble, at temperature corresponding to the Hawking temperature of the black hole, or alternatively in the microcanonical ensemble at fixed energy.

The conformal field theory is unitary and can be viewed as living on the left conformal

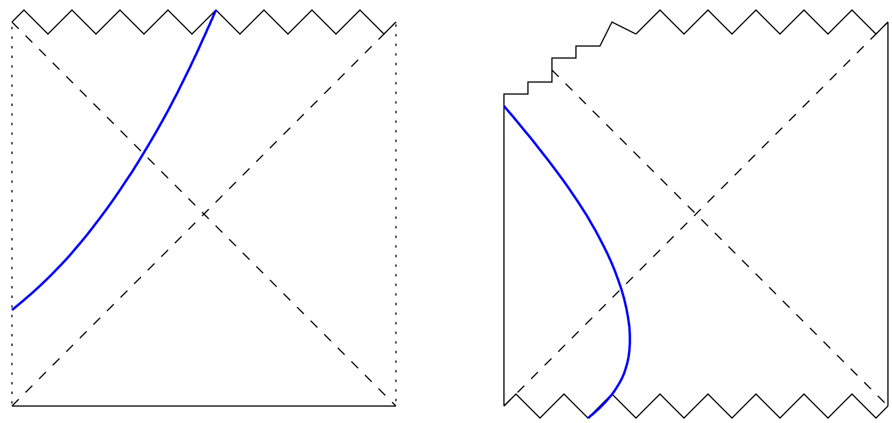

Figure 4: Penrose diagrams after taking into account perturbations. The left diagram shows the bubble trajectory in dS spacetime. The region to the right of the bubble wall is excised. Eventually any given worldline tunnels back into an AdS region, leading to a big crunch. The diagram on the right shows the trajectory through AdS spacetime. The region to the left of the wall is excised. Perturbations lead to a Cauchy horizon with a null singularity meeting the bubble wall at infinity.

boundary $\mathbb{R} \times S_2$. Since the spatial directions are compact the CFT entropy in these statistical ensembles will be finite and given by the Bekenstein-Hawking entropy of the black hole S_{BH}. For timescales of order the Poincare recurrence time $e^{S_{BH}}$, the correlators will deviate wildly from the thermal correlators [23–28], but ultimately will settle down again to quasi-thermal correlators. This is represented in figure 5 by singularities separating well-behaved semiclassical regions with a SAdS black hole exterior.

Note that some of these regions apparently have two disconnected boundary regions at infinity [29] (such as the second region from the top in figure 5). However, in line with the setup in the previous paragraphs, these asymptotic regions are not to be viewed as completely independent of each other, but rather as representing analytic continuation of modes with finite energy from a single exterior region, dual to states in a single CFT. Ultimately these bulk geometries are approximate descriptions of the time evolution of a pure state in a single CFT.

In general states emerging from the white hole singularities in figure 5 will contain all excitations present in the thermal ensemble. In particular, the bubble solutions of section III will appear with nonvanishing probability. Since the Poincare recurrences go on forever, we have an infinite number of times to produce bubbles with realistic cosmological interiors,

including inflation, while at any given time only a finite number of CFT states need be considered. These transitions populate the string theory landscape in a *timelike* manner instead of a *spacelike* manner as in standard eternal inflation [30–37]. We refer to this scenario as *punctuated eternal inflation.* As we will see, many of the attractive features of eternal inflation survive in this picture, while at the same time, problems of the cosmological measure and many other issues are ameliorated.

B. Bubble states in the CFT

It is helpful to begin by first considering modes of a scalar field with finite global energy, and the relevant vacuum state. Let us review Unruh's construction [38] adapted to the eternal SAdS geometry. The geometry includes two asymptotic regions related via analytic continuation. Any finite frequency Kruskal modes can be continued analytically across the horizon and used to provide a complete basis in the full SAdS manifold. Likewise, Rindler modes can be patched together to define global modes. For the Rindler modes, positive frequencies are defined with respect to asymptotic timelike killing vectors, and positive frequency for global modes is defined with respect to the null killing vector at the horizon when the metric is written in Kruskal-like coordinate[38].

The two types of vacuum state are related by

$$|0\rangle_{Kruskal} = \prod_{\omega} \exp(e^{-4\pi GM\omega}\, b^{\dagger}_{1\omega} b^{\dagger}_{2\omega})|0\rangle_1 \times |0\rangle_2$$

where $1, 2$ indicate the left/right asymptotic regions of SAdS and $b_{1/2}$ ($b^{\dagger}_{1/2}$) are the respective Rindler annihilation (creation) operators. The Kruskal annihilation operator can expressed in terms of the creation and annihilation operators on the left and right regions as follows,

$$a^{\dagger Kruskal}_{\omega} = b^{\dagger}_{1\omega} + e^{-4\pi GM\omega} b_{2\omega}\,. \tag{4}$$

Thus we see that a finite energy mode well-localized in one asymptotic region will necessarily have an exponentially suppressed component in the other asymptotic region. These finite frequency bulk modes may then be mapped to operators in the conformal field theory via the standard bulk to boundary map of [39, 40], generalized to the eternal black hole background.

Now we have so far made the convenient approximation that the bubble wall is thin. The

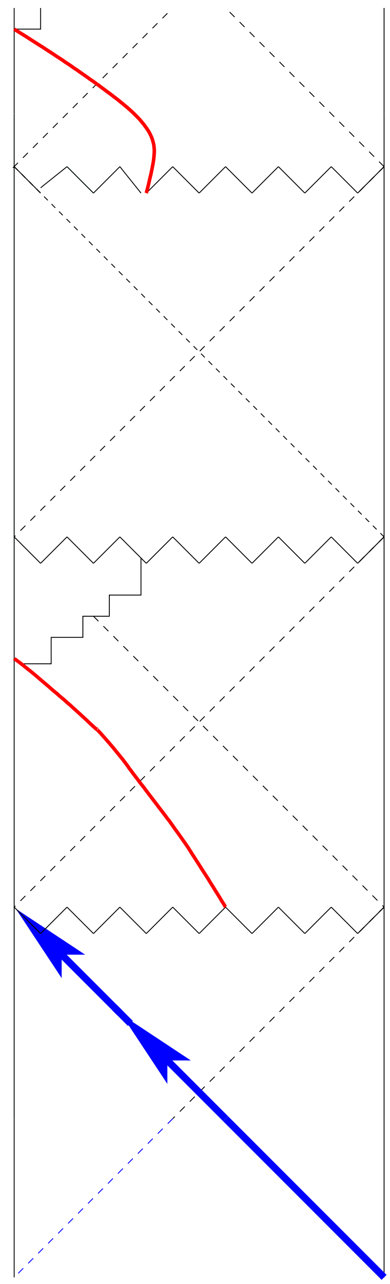

Figure 5: Punctuated Eternal Inflation: Starting with empty AdS space, dual to the vacuum state of the CFT, a light-like shell, shown by the blue line, collapses into a black hole. A bubble with dS interior, shown by the red curve, appears out of the spacelike singularity. The left side of the red line i.e. the bubble wall is a piece of dS space. This sequence repeats, following the quasiperiodic Poincare recurrences in the dual gauge theory.

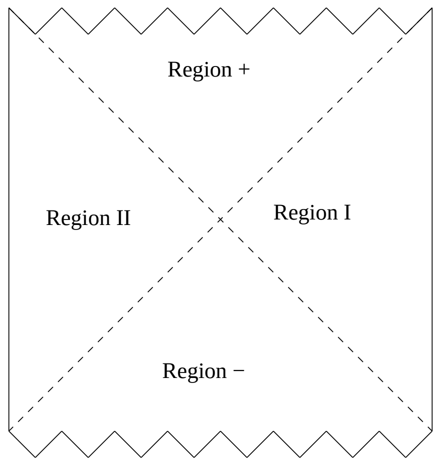

Figure 6: The two asymptotic regions of SAdS

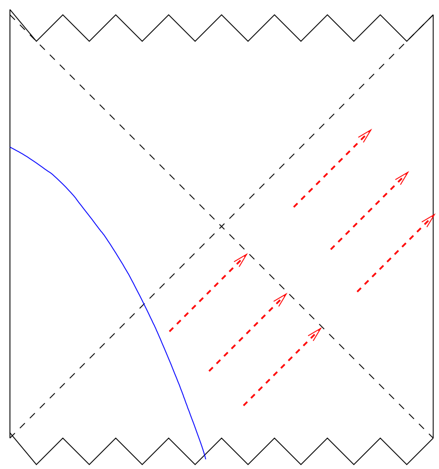

Figure 7: Radiation coming off the bubble walls

essential details of the construction are not expected to change if we generalize to consider walls of finite thickness, that might be built from finite frequency modes. Moreover, we expect operators creating and annihilating bubbles to obey a similar equation to (4). This is good news for representing such operators in the CFT, since they may be reconstructed using the positive and negative frequency modes in only the right asymptotic region, which may then be mapped into the CFT. In addition to this leading order effect, further details of the bubble state can be extracted from the radiation coming off the bubble wall to the left asymptotic region (depicted by the red dashed arrows in the figure 7).

To study the effects of the radiation coming off the bubble we model the bubble wall as a perfect mirror i.e. impose purely reflecting boundary conditions for the (scalar) field propa-

gating in the SAdS background and restrict ourselves spherical symmetry. The computation is sketched in Appendix B. Further specification of the bubble would require the particular details of the metastable string theory vacuum giving rise to the potential shown in figure 1. Nevertheless, for a given potential and choice of initial state in the gravity dual, one has a well-defined (one-to-many) mapping into the CFT.

At this point let us collect a few facts on the spectrum of bubble states, which will be useful in the subsequent sections. One could compute frequencies of normalizable excitations around the bubble geometry by solving the gravity equations of motion. However this spectrum is expected to receive important corrections due to back-reaction and quantum effects. The CFT enables us to take these effects into account. Even without all the details of the CFT dual that incorporates a bulk potential of the form figure 1, some qualitative features of the spectrum can be determined. The domain wall states considered in the present scenario are viewed as a subset of black hole microstates. These are dual to states in the CFT_3 on the spacetime $\mathbb{R} \times S_2$, therefore the entire spectrum of CFT states will be discrete. We can schematically write the microcanonical density matrix to be,

$$\rho(E) = \sum_{a,b,\ldots} |\psi_{a,b,\ldots.}\rangle\langle\psi_{a,b,\ldots}|$$

where

$$\mathbf{H}|\psi_{a,b,\ldots.}\rangle = E|\psi_{a,b,\ldots.}\rangle$$

are energy eigenstates with *discrete* internal labels $a, b, \ldots.$. For large total energy E this can be well-approximated by the density matrix of the canonical ensemble

$$\rho(\beta) = \sum e^{-\beta\mathbf{H}}|\psi_{a,b,\cdots}\rangle\langle\psi_{a,b,\cdots}|$$

where β is the black hole inverse temperature. This may then be directly mapped into the canonical ensemble in the CFT at inverse temperature β. This will be our basic working definition of the ensemble of states that contains inflating bubbles of the type shown in figure 7. As we will see, this is already sufficient to provide interesting bounds on the measures of cosmological interest.

Of course the canonical ensemble also contains many states that do not represent cosmological solutions of interest. One could construct a more refined ensemble by selecting on black holes with expanding interiors. Local bulk correlators can be constructed in these

spacetimes (including points inside the bubble) by generalizing the two-dimensional construction of [19]. Correlators of massless fields will exhibit a characteristic powerlaw falloff with geodesic separation inside the bubble, and this geodesic separation depends directly on the vacuum energy inside the bubble. Using the methods of [16–18] these correlators may be mapped into integral transforms of correlators in the CFT. This yields a straightforward, though calculationally tedious way of selecting the vacuum energy inside the bubbles.

Another, more direct way, to select on expanding bubble states is to use characteristic radiation coming off the bubble (as shown in figure 7) to refine the specification of the CFT state. As mentioned above, this is explored in a simplified model in appendix B. On the gravity side the solution is that of a black hole with a set of quasinormal modes excited. The amplitudes for these modes will be determined by the bubble initial state, and the detailed properties of the potential of figure 1. This can be represented by a time-dependent density matrix in the CFT, that will, in general, depend on the vacuum energy inside the bubble.

V. CONSISTENCY CHECKS

The semiclassical consistency hypothesis of section II by construction produces quantum versions of de Sitter space with a sensible semiclassical limit as $\hbar \to 0$ and $S_{GH} \to \infty$. In this section we perform some consistency checks on the approach to the semiclassical limit, with a view to keeping S_{GB}, S_{BH} and $\hbar$ finite to use as a physically sensible regulator for eternal inflation. The detailed scenario we have in mind for the remainder of the paper is shown in figure 1, where a worldline begins in a rapidly inflating phase, that later settles into a metastable de Sitter vacuum, that will be modeled as the late-time development of bubbles solutions such as those shown in figure 7.

We begin by studying the late-time history of this worldline. From the original work [22, 41] we know that a worldline in the dS bubble tunnels back to the stable AdS vacuum. This event is accompanied by extra excitation energy that quickly leads to a big crunch. Related tunneling solutions within the AdS/CFT context have recently been studied in [42]. From an estimate of the tunneling rate we can we know how long this worldline lasts in proper time. This time is (see, for example [5])

$$\tau_{decay} \sim e^{S(\phi)} e^{S_{GH}}, \tag{5}$$

where $S(\phi) < 0,\ |S(\phi)| \gg 1$ and is just the $O(4)$ invariant Euclidean action for the tunneling trajectory from dS to AdS space. This may be converted into a timescale in the CFT by mapping to Schwarzschild time in the exterior AdS region. This calculation is performed in appendix A, eqn. (A2).

$$t_{decay,SAdS} \approx B - \frac{Al_{AdS}^2}{l_{dS}} e^{-e^{cS_{GH}}},\ 0 < c < 1$$

where A, B are constants defined in appendix (A). Thus the decay time is typically shorter than a Planck time in SAdS coordinates, from the moment when the bubble wall reaches the null singularity in the exterior region, as shown in figure (4). For such short timescales we should not trust the semiclassical gravity description, though the CFT description is valid. We therefore conclude that this tunneling process happens long after conventional semiclassical physics has broken down in the dS bubble.

The Poincare recurrence time for de Sitter space $\tau_{Poincare,\,dS} \sim e^{S_{GH}}$ is already much longer than the tunneling timescale (5). This eliminates all the issues one encounters when the decay time exceeds the recurrence time, for instance breakdown of a classical general relativity by reversal of the arrow of time in a semiclassical regime as discussed in [43].

The finest time resolution we can reasonably hope to resolve in the bulk is

$$\Delta t_{SAdS} = l_{pl}$$

the Planck time. Translating this back into de Sitter proper time (as shown in appendix A) we obtain

$$\tau_{dS} = l_{dS} \log\left(\frac{Al_{AdS}^2}{l_{pl} l_{dS}}\right). \tag{6}$$

As we will later see, this timescale will typically be $l_{dS}10^d$ where d is some number of order 1. Thus we see that in this scenario our semiclassical de Sitter bubble survives for a far shorter than one might have imagined based on tunneling probabilities, before the discreteness of the CFT makes its presence felt. Nevertheless, as we will see, for reasonable parameter choices, our present cosmological history can be embedded in this scenario.

Now let us address the question of whether for times smaller than (6) the CFT can accurately reproduce correlators inside the dS bubble. This issue arises in studies of the black hole information problem, where the CFT must encode the details of the internal state of the black hole. As shown in [44, 45], for effective field theory in black hole backgrounds, the

departure from locality on observations made over times less than the scale in Schwarzschild coordinates $\Delta t_{SAdS} \sim S_{BH}\beta_H$ is less than the scale $e^{-S_{BH}}$ (for example the amplitude of Hawking emission) following information theoretic arguments first invoked in [46]. Recall that the AdS$_5$/CFT$_4$ map implies $S_{BH} \propto N^2$ and hence $e^{-S_{BH}} \sim \frac{1}{e^{N^2}}$ represents effects which are non-perturbative in $1/N$ in the dual gauge theory. For the $d = 3$ case the dual CFT is expected to be a generalization of the CFT studied in [47]. There the CFT was realized as a Chern-Simons theory with Chern-Simons level number k playing the role of gauge coupling, g^2_{YM}.

The observer in the dS bubble will only be able to measure, for example, energy with an intrinsic error of $1/l_{dS}$ given the above discussion. This is already a much larger error than the error potentially allowed from a construction of bulk amplitudes from the exact CFT. Therefore we conclude there are no immediate obstructions to reproducing semiclassical physics inside the bubble from the exact CFT description, for proper times less than (6).

VI. IMPLICATIONS FOR COSMOLOGY

In this section we examine a number of standard issues in cosmology within this scenario.

A. Bound on the number of e-foldings of inflation

Suppose we make a bubble with $\Lambda_{dS} = \Lambda_{today} \sim (10^{-3}eV)^4$ and say it developed from a GUT scale bubble excitation $\Lambda_{GUT} = (10^{16}GeV)^4$ following the time evolution sketched in figure (1). Let us denote the maximum number of e-foldings in the high-scale phase by n_{max}. If we use the Gibbons-Hawking entropy to count states in the different de Sitter phases, then balancing the entropy before and after high-scale inflation, we have

$$\begin{aligned} \text{No. of Hubble volumes } \times S_{GUT} &\leq S_{today} \\ n_{max} &\leq \frac{1}{3}\log\frac{\Lambda_{GUT}}{\Lambda_{today}} \\ n_{max} &\leq 86. \end{aligned} \tag{7}$$

So we see the present scenario reproduces the bound on the number of efolds derived in [48]. These kinds of bounds are discussed further in [49].

The logic presented here is rather different to that of [48]. Here we match the number of available microstates within the ensemble of states that evolves toward a single large Λ_{today} bubble to derive the bound. In [48] they instead consider the entropy of a fluid in de Sitter and match the entropy of the stiffest fluid, that of a gas of black holes, in order to reach the bound (7). Nevertheless it is satisfying the bound of [48] is reproduced when de Sitter spacetime is embedded in a complete unitary theory of quantum gravity, such as the AdS/CFT duality. The present scenario offers a number of modifications and refinements to the *Cosmological Complementarity* principle presented in [50].

B. Natural minimization of the cosmological constant

A fundamental puzzle in modern cosmology is smallness of the cosmological constant compared to the scales of fundamental physics. In terms of Hubble parameters, there is a huge hierarchy

$$\frac{H_{today}}{H_{inflation}} \propto \sqrt{\frac{\Lambda_{today}}{\Lambda_{GUT}}} \sim 10^{56} \, .$$

The present scenario gives us a natural explanation for decrease of the cosmological constant. Let us consider the ensemble of microstates that can evolve to a single large bubble, which we will refer to as the grand ensemble. For timescales smaller than (6) we treat this ensemble as a closed system. Semiclassical evolution of the bubble implies that it keeps growing for as long as possible. The classical singularity theorems of Hawking and Penrose [51] imply the initial state began at high density. As the bubble expands, the number of states is ultimately bounded by $e^{S_{BH}}$, the number of black hole microstates in the external AdS spacetime.

We model the GUT-scale phase of inflation by a similar ensemble with $\Lambda_{dS} = \Lambda_{GUT}$, which we refer to as the small ensemble. With each e-folding an entire Hubble volume worth observables are added to the small ensemble. We should view the GUT-scale region as contained within the grand ensemble of the previous paragraph, since ultimately it evolves toward it. Therefore we can apply the second law of thermodynamics to the small ensemble and find that the entropy (the log of the number of available microstates) increases as the bubble expands. This, of course, can only be consistent if $\Lambda_{today} < \Lambda_{GUT}$. Extending this argument by relating the cosmological entropy in a background with time-dependent Hubble

parameter [52] to the microscopic entropy, one finds

$$\dot{S} \geq 0 \Rightarrow \dot{H} \leq 0\,.$$

A closely related argument has been put forward by Strominger [53] based on a *c−theorem* by viewing the de Sitter regions as dual to CFTs. He proposed our universe is represented as a renormalization group (RG) flow between two conformal fixed points where the forward time-translation in the bulk was interpreted as an IR-to-UV RG flow of a putative dual CFT. The Hubble parameter,

$$H = \frac{\dot{a}}{a} \tag{8}$$

was proposed to play the special role of the (inverse of) central charge of the dual CFT,

$$c = \frac{1}{HG} \tag{9}$$

because for matter obeying the null energy condition, the Einstein equation implies,

$$\dot{H} \leq 0 \Rightarrow \dot{c} > 0\,.$$

In the context of the present scenario, the conformal symmetry of the de Sitter phase arises as an emergent symmetry of the subset of black hole microstates corresponding to the expanding bubbles, over a restricted range of timescales. Nevertheless, because the gravity problem can be understood in terms of a RG flow, it suggests that these de Sitter solutions do correspond to stable (or at least quasi-stable) fixed points. It remains an open problem how to better identify these fixed points more directly once embedded in the consistent AdS/CFT framework.

C. Unitarity and eternal inflation

One of the features of the standard eternal inflation scenario is that once started, there is always a region on a given spacelike slice where high-scale inflation is occurring. Essentially baby universes branch off in these highly quantum regions. It is difficult to imagine how such processes can preserve unitarity in a quantum description. If the number of microscopic degrees of freedom changes with time, then unitarity is necessarily violated. Certainly if

such branching processes occurred locally, we can invoke the results of [54] that show that such violations of unitarity would lead to Planck scale violations of energy and momentum conservation in the bulk.

Aside from the issue of unitarity, standard eternal inflation has the attractive feature that the probability distribution of inflating regions should eventually reach a stationary state. This has the potential to lead to some very interesting predictions, however as we will review later, the infinite nature of the spacelike slices in this stationary state has so far stymied this effort.

Another useful feature of eternal inflation is that it provides a natural mechanism for populating the landscape of string theory vacua. There is always some region on a given spacelike slice in a position to roll down to one of the stable or metastable vacua of string theory. This phenomena then opens the door to anthropic arguments playing a role in our understanding of the observed parameters of nature.

In the bubble scenario, unitary is no longer an issue, since the states are realized as states in a unitary conformal field theory. As we have seen, this regulates both the spacelike and timelike extent of a cosmological region that may be treated semiclassically. However as far as we have seen, there is room to describe a large expanding universe for a large number of Hubble times.

As we have seen, the semiclassical consistency hypothesis of section (II) rules out tunneling to semiclassical expanding bubbles. Because baby universes cannot form from smooth initial data, the problems with bulk unitarity described in [54] are avoided. Instead expanding bubbles only emerge from punctuation points, i.e. the spacelike singularities of black/white holes. Unlike standard eternal inflation, a time-independent probability distribution does not emerge.

However Poincare recurrences in the CFT lead to quasi-periodic recurrence. Thus rather than in the spacelike direction, we have infinite extent in the timelike direction. The randomness of the recurrences can provide a mechanism to populate whatever stable and metastable vacua are present, so this attractive feature of eternal inflation is preserved.

D. Measure on the space of initial data

Now as emphasized before we can work in a microcanonical ensemble since the spacetime goes through many *punctuated* phases and has sufficient time to explore the entire phase space. So at the fundamental level, the measure on the space of initial data should be set by ergodicity. In particular, all bubble states at equal energy will be equally probable. The precise probability distribution will depend on the details of the spectrum of black hole microstates in the particular CFT dual to the gravity background with potential shown in figure 1. However a number of general properties of these distributions can be determined simply from a knowledge of the black hole entropy, which must bound the bubble entropy, and the interpretation of bubble entropy described in section II and in [13].

For single bubbles, one could approximate the probability distribution as

$$P_{bubble}\,(l_{dS}) = \frac{e^{S_{GH}(l_{dS})-q(l_{dS},M)}}{e^{S_{BH}(M)}}, \tag{10}$$

where M is the fixed mass of the large AdS black hole that dominates the microcanonical ensemble, and $q(l_{dS}, M) \geq 0$ represents a kind of form factor that parameterizes the likelihood of the black hole having such bubble solutions as microstates. Qualitatively, we expect $\lim_{l_{dS}\to l_{pl}} q(l_{dS}, M) = 0$ since the formation of small bubbles near the singularity should be a local process, independent of M. However $\lim_{l_{dS}\to r_{BH}} q(l_{dS}, M) \gg 1$, where r_{BH} is the black hole horizon radius, reflecting the fact that no bubble states larger than the black hole are allowed. This means large single bubbles are more likely, with a typical size $l_{pl} < l_{dS} < r_{BH}$.

Now let us consider the effect of multiple bubbles. For very large bubbles, only a single bubble will fit inside the black hole, so the question becomes whether there is more entropy in a soup of small bubbles. Let us approximate the collection of small bubbles by a gas of particles in thermal equilibrium with the anti-de Sitter Schwarzschild black hole. The entropy of this gas will be bounded above by the entropy of a gas of massless particles (which for the sake of simplicity we can take to be conformally coupled). This one of the systems studied in the work by Hawking and Page [55]. They find that for $M > m_{pl}^2 l_{AdS}$ that the black hole entropy dominates over the entropy of the massless particles. Here $m_{pl}^2 = 1/G$, with m_{pl} the Planck mass. Assuming the form factor $q(l_{dS}, M)$ does not fall off very rapidly, the entropy of single large bubbles will therefore dominate over that of multiple small bubbles, for sufficiently large black holes. The probability distribution for different values of l_{dS} can then be well-approximated by (10) assuming the gravity theory has access

to a full landscape of string theory vacua with many different values of l_{dS} possible, so that $q(l_{dS}, M)$ is a smooth function.

E. Resolution of cosmological paradoxes

In the present scenario time development is future eternal, and at any given time the region along a spacelike slice undergoing interesting cosmology (i.e. a large region in an expanding phase) is of finite extent. It will be useful to count the number of potential observers within a given causal diamond inside this region. To do this we compute the entropy per Hubble volume inside the expanding bubble as a function of time. Here we have in mind a bubble with an interior rolling down the potential as shown in figure 1, so for now on we will generalize to a Robertson-Walker metric for the interior

$$ds^2 = -dt^2 + a(t)^2(dr^2 + d\Omega^2)\,.$$

Denoting the densities by ρ_ψ, ρ_γ and ρ_Λ for the densities of matter, radiation and cosmological constant respectively, the Friedman equations are

$$\dot{a}(t) - a(t)\sqrt{\frac{8\pi l_{pl}^2}{3}\left(\rho_\psi + \rho_\gamma + \rho_\Lambda\right)} = 0$$

$$\dot{\rho}_\psi(t) + 3\frac{\dot{a}(t)}{a(t)}\rho_\psi(t) = 0$$

$$\dot{\rho}_\gamma(t) + 4\frac{\dot{a}(t)}{a(t)}\rho_\gamma(t) = 0$$

$$\dot{\rho}_\Lambda(t) = 0\,.$$

The entropy density for any component is

$$s_i = \frac{(\rho_i + P_i)}{T_i} = \frac{(1+w_i)\rho_i}{T_i}$$

where $P = w\rho$ is the equation of state ($w = 0, \frac{1}{3}, -1$ for matter, radiation and, cosmological constant respectively). Now the temperature of each component is given by,

$$\frac{1}{T_i} = \frac{\partial S_i}{\partial E_i} = \frac{\partial s_i}{\partial \rho_i}$$

and the entropy density as a function of the ρ is then,

$$\frac{s_i}{s_{i0}} = \left(\frac{\rho_i}{\rho_{i0}}\right)^{\frac{1}{1+w_i}}$$

and similarly the energy density of each component is,

$$\frac{\rho_i}{\rho_{i0}} = \left(\frac{a(t)}{a_0}\right)^{-3(1+w_i)} .$$

So the entropy per Hubble volume from matter and radiation is,

$$S_H(t) = \sum_i s_i \left(\frac{1}{H}\right)^3 = s\left(\frac{a_0}{\dot{a}(t)}\right)^3 \tag{11}$$

where $s = \sum_i s_{i0}$ and we sum over only matter and radiation components, and take this as a measure of the number of potential observers in the bubble as a function of time. In addition, there is a small contribution from Hawking radiation associated with the cosmological horizon

$$S_\Lambda = \frac{\pi^2}{15} T_{dS}^3 \frac{1}{H_{dS}^3} \sim O(1) . \tag{12}$$

This component is constant with time, and dominates at late times . This leads to the issue of so-called Boltzmann Brain observers, that we discuss momentarily.

The entropies are shown in figure 8. The peak of the entropy curve is located at $\ddot{a} = \sum_i (\rho_i + 3P_i) = 0$ i.e. at

$$\rho_\psi + 2\rho_\gamma - 2\rho_\Lambda = 0 \tag{13}$$

which typically happens for $t < l_{dS}$ when there is a balance between matter, radiation and cosmological constant energy densities. This will be less than the timescale t_{sc} when semiclassical physics breaks down (6) if we have a region where gravity is a good description inside the bubble. Also shown on the plot is the timescale when Boltzmann brains dominates t_{BB}, which will always be much larger than the timescale (6). This figure will play a key role in seeing how various cosmological paradoxes are resolved.

1. *Cosmological measure*

The theory of inflation has had many successes in terms of explaining a number of observed phenomena within our Hubble volume, such as providing natural explanations of the

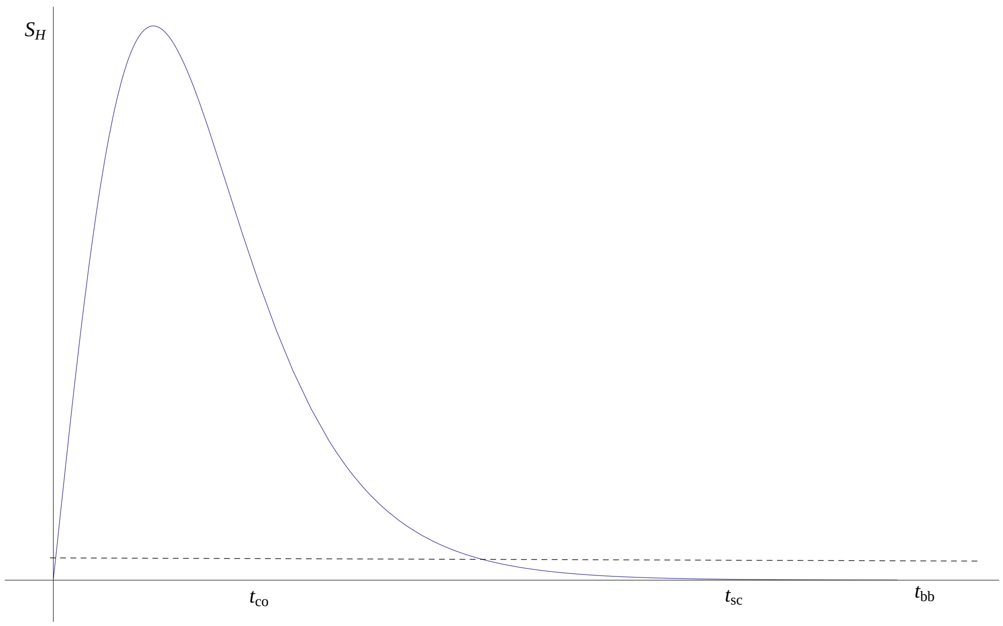

Figure 8: Entropy of matter and radiation per Hubble volume as a function of time. This provides an estimate of the number of potential observers as a function of time.

horizon problem, the flatness problem and the spectrum of cosmic microwave background perturbations. Extending the theory of inflation using semiclassical approaches to quantum gravity leads to eternal inflation [30–37], which describes a multiverse of pocket universes. The goal then becomes how to ascribe probabilities to histories through this multiverse, i.e. the cosmological measure problem.

In eternal inflation the "pocket universes" are infinite in number. Without any regularization, probabilities, being a ratio of two infinities, are ill-defined. Even after regularization there is no guarantee that the answer could not depend on the regularization method. Current attempts to define the measure tend to be coordinate dependent and and hence ambiguous ([56] and references therein).

This issue is resolved in the present scenario: a natural regulator that preserves unitarity is introduced. A probability measure for different cosmologies can then be extracted by evaluating (10) for a given conformal field theory, which computes the likelihood of expanding bubbles as a function of their parameters.

As emphasized in [57, 58] what is most relevant for observation are probabilities conditioned on the existence of observers. This can be incorporated into the discussion in the following way. The density of observers can be expected to be proportional to the entropy

density of matter (and perhaps also radiation). However entropy density alone is not a good guide, because one could have a high density, rapidly expanding region of spacetime. In this case the number of degrees of freedom in causal contact with each other can remain small, so complex systems, such as observers, will not form.

Therefore we propose to count the amount of matter and radiation per Hubble volume as an approximate measure of the number of degrees of freedom in causal contact with each other. This has the advantage of being a locally defined quantity along a given worldline. Moreover it reduces to the volume of a causal diamond when the cosmological constant dominates the expansion.

The result is a world-line measure that determines the likelihood an given observer will measure a particular set of landscape parameters, as well as other cosmological observables. For example, if we condition on the the observer being in an expanding region where semi-classical gravity operates, then we conclude from (10) and figure 8 that the most likely observer will exist at time $\tau = \tau_{peak}$.

2. Cosmic Coincidence Problem:

Another fundamental puzzle of cosmology is why the energy density of dark energy is comparable to the energy of the matter today

$$\rho_\Lambda \sim \rho_{Matter} \, . \tag{14}$$

This is resolved by the world-line measure described in the previous section. A typical observer will appear near the peak of the entropy per Hubble volume, where the densities are related by (13).

This computation is similar in spirit to the argument of [59]. There they generalize Weinberg's argument [58] for the most likely cosmological constant and treat both the cosmological constant and the density contrast at the time of recombination are random variables. They conclude that most galaxies form at the time of cosmological constant dominance, providing an anthropic explanation of (14). The world-line measure of the previous section provides a much simplified version of this computation, relying only on $H(t)$.

3. *Youngness Paradox and Oldness Paradox*

One of the more popular cosmological measures in standard eternal inflation assigns likelihood proportional to volume of a given pocket universe. In this picture, a region of large vacuum energy expands very rapidly and comes to dominate the volume along a family of spacelike hypersurfaces (see for example [56]. Thus for any finite time truncation the pockets with Planck scale expansion dominate in an overwhelming manner. This would then make our universe which has been around for billions of years a very special old pocket [36].

The cosmological measure described in section VI E 1 also resolves this paradox. A typical bubble will actually be one with a very large number of states according to (10). Since this state evolves from a singular state, an observer will see higher density looking toward the past, but such an observer will typically sit near the coincidence point (13).

4. *Boltzmann Brains*

If the bubble were to be arbitrarily long-lived, it might happen that the area under the curve in figure 8 associated with Hawking radiation could dominate, versus the area under the matter/radiation peak. This leads to a potential oldness paradox in the present scenario, which is often referred to as the Boltzmann Brain paradox.

In the usual picture of eternal inflation expansion is eternal. This leads to Boltzmann brain observers that form at late times from thermal or vacuum fluctuations (see for example [43]). Again volume weighting of the expanding regions leads to an infinite number of Boltzmann brains in comparison with ordinary observers living around the time of cosmic coincidence. Since Boltzmann brains would infinitely dominate over ordinary observers, observations like ours would be atypical. Again a suitable solution to the measure problem is needed to cure the putative catastrophe of Boltzmann brains.

In the present scenario, semiclassical physics breaks down long before Boltzmann brains become an issue, and the problem is avoided. If the entropy of a Boltzmann brain is $S_{BB} \gg 1$ the timescale for formation will be of order

$$\tau_{BB} = l_{dS} e^{S_{BB}} \,. \tag{15}$$

This is to be compared with the timescale at which semiclassical physics breaks down (6).

Having a separation of scales between l_{dS} and l_{pl} so that semiclassical physics is valid requires some fine tuning, but these scales appear inside a log in (6). A much higher degree of fine tuning is needed to make this time larger than (15). Therefore most worldlines will not see dominance of Boltzmann brain observers.

If one does manage to find a region of the landscape where (15) is less than (6), one still avoids eternal expansion of the bubble because the worldline eventually tunnels back to the stable AdS region as described in section V on timescale given in (5). We conclude that while there may be regions of the landscape where Boltzmann brain observers could dominate, extreme fine tuning is required. A detailed specification of the CFT would be needed to make sharper statements.

In the scenario where an expanding bubble lives inside an exterior SAdS geometry, there is a different type of Boltzmann brain that can appear in the exterior region. These are potentially more dangerous. Consider the history of such an observer who is created by a thermal fluctuation outside the Schwarzschild black hole. Such an observer will follow a timelike geodesic, and will enter the black hole horizon and hit the singularity on a timescale of order r_{bh}. Semiclassical gravity should be a good approximation in this exterior region, and modulo Poincare recurrences, this region is eternal. Therefore we expect a substantial number of such observers to appear and this number could overwhelm the number of ordinary observers.

In this scenario, the stable AdS vacuum state is supersymmetric, so atoms and other complex bound states will not appear. Thus the kind of complex quasi-stable bound state corresponding to a Boltzmann brain may simply not appear in this region. Rather such an observer would likely need a large supersymmetry breaking bubble to surround her, which leads us back to the original scenario.

Alternatively, for observables of interest to us, we can condition probabilities by the requirement that observers live in an expanding phase. This rules out this dangerous class of Boltzmann brain observers. While it is straightforward to impose this condition in the limit $l_{pl} \to 0$ where semiclassical gravity is valid, it remains to be seen whether this distinction is possible in the full quantum regime.

5. Problems of time

The time coordinate is not a physical observable in a diffeomorphism invariant formulation of gravity. Usually the way around this is to use a rolling scalar field to define an invariant notion of time. However this then raises issues with recovering locality and causality. These issues are reviewed in [60].

In the AdS/CFT context, time on the conformal boundary provides a global time coordinate. The CFT comes equipped with a well-defined norm, and time evolution in the conformal field theory is unitarity. The CFT norm induces a well-defined measure on the space of bubble cosmologies, of which (10) is a simple example. Less clear is how bulk locality and causality emerges, but this is a topic under active investigation.

Another problem of time is how the arrow of time arises in a CPT invariant theory such as gravity. Let assume the history shown in figure (5) effectively explores the microcanonical ensemble in the CFT. In addition to expanding bubble states, by CPT invariance we will also have an equal number of time-reversed collapsing bubble states. However any given macroscopic bubble solution that gives an expanding or contracting region de Sitter region will necessarily be time asymmetric [13]. Thus in this scenario the arrow of time arises from spontaneous symmetry breaking: the underlying theory is CPT invariant, but the solutions break this symmetry.

Acknowledgments

The research of D.A.L. and S.R. is supported in part by DOE grant DE-FG02-91ER40688-Task A. S.R. is supported in part by Lehman College, City University of New York.

Appendix A: Mapping global time to bubble proper time

In this appendix we find the relation between comoving time inside the dS bubble, and Schwarzschild time in the AdS region. The proper time along the bubble wall τ satisfies (1) with the radial coordinate $r(\tau)$ determined by (2). This allows us to determine $t_{SAdS}(r)$ at the bubble wall via

$$dt_{SAdS} = dr\left(\frac{1}{f_{SAdS}(r)}\left(\frac{1}{f_{SAdS}(r)} - \frac{1}{V_{eff}(r)}\right)\right)^{1/2} .$$

Integrating in the approximation, $r \to \infty$,

$$t_{SAdS}(r) = B - A\frac{l_{AdS}^2}{r}, \quad A = \sqrt{1 + \frac{4\kappa^2 l_{AdS}^2 l_{dS}^4}{(l_{AdS}^2 + l_{dS}^2(\kappa^2 l_{AdS}^2 - 1))^2 + 4l_{AdS}^2 l_{dS}^2}} \tag{A1}$$

with B an integration constant. We see the bubble reaches $r = \infty$ at finite Schwarzschild time $t_{SAdS} = B$.

In the interior, the r coordinate becomes timelike for $r > l_{dS}$, while t_{dS} approaches a constant on the bubble wall $t_{dS} = C$. Now let us transform to comoving time τ_{dS} in the de Sitter patch, working with flat spatial slices

$$ds^2 = -d\tau_{dS}^2 + l_{dS}^2 e^{2\tau_{dS}} d\vec{x}^2 .$$

The comoving time τ_{dS} is related to r by

$$\tau_{dS} = l_{dS} \log\left(\frac{r}{l_{dS}}\right)$$

so inserting this into (A1) we obtain

$$t_{SAdS} = B - \frac{A l_{AdS}^2}{l_{dS}} e^{-\frac{\tau_{dS}}{l_{dS}}} . \tag{A2}$$

Now the semiclassical approximation is not expected to continue to hold very close to the null singularity induced by back-reaction(4). If we assume the semiclassical approximation holds to $t_{SAdS} = B - l_{pl}$ this translates into breakdown at comoving de Sitter time

$$\tau_{dS} = l_{dS} \log\left(\frac{A l_{AdS}^2}{l_{pl} l_{dS}}\right) .$$

Appendix B: Modeling the bubble signature

In section IV B, the signatures of the bubble state in the CFT are described. One such signature is radiation propagating through the bubble wall and out to infinity in the AdS region, which can be related straightforwardly to CFT correlators. In this appendix, we develop a simple model for this radiation signature. Working in the semiclassical probe approximation we consider a massless (conformal) scalar field propagating in this bubble

geometry with the purely reflective boundary conditions at the bubble wall and zero flux at the AdS spacelike infinity. For computational convenience we specialize to $(1+1)$-d. Since all space times are conformal to flat (Minkowski) space time in $(1+1)$-d, this is conformally mapped to the propagation of a scalar field in a flat background with moving mirror [61].

Introduce null coordinates u, v on the flat spacetime

$$ds^2 = dudv\,.$$

The moving mirror boundary follows the trajectory $x(u), \tau(u)$ where $u \equiv \tau(u) - x(u)$. The modes solutions are,

$$\phi_\omega = e^{-i\omega v} + e^{-i\omega p(u)}$$

where

$$p(u) = 2\tau(u) - u.$$

In the mirror rest frame $(U{,}V)$ with mirror at the origin,

$$V = v, \qquad U = p(u)$$

or,

$$ds^2 = dudv = \frac{1}{p'(u)}\,dUdV\,.$$

Performing a conformal transformation, the energy flux in u, v frame [61], is

$$T_{uu} = \frac{1}{12\pi}\sqrt{p'}\partial_u^2\left(\frac{1}{\sqrt{p'}}\right). \tag{B1}$$

Now lets move on to pure AdS_2. The metric is,

$$\begin{aligned} ds^2 &= -(1+r^2)dt^2 + \frac{dr^2}{1+r^2}, \quad t, r \in (-\infty, \infty) \\ &= -\sec^2 x\,(dt^2 - dx^2),\ x \equiv \tan^{-1} r \in [-\pi/2, \pi/2] \\ &= -\sec^2\frac{v-u}{2}\,dudv, \quad u, v \equiv t \mp x\,. \end{aligned}$$

As before we now shift to a frame where the mirror is at rest at the origin (i.e. $U = p(u)$ and $V = v$) and we can (conformally) relate this to the mirror at rest in flat space case,

$$ds^2_{AdS} = -\sec^2\frac{v-u}{2}dudv = -\sec^2\left(\frac{V - p^{-1}(U)}{2}\right)\partial_U p^{-1}(U)\,dUdV = C(U,V)dUdV\,. \tag{B2}$$

Then the renormalized energy-momentum tensor expectation in the U, V vacuum is,

$$T_{UU} = -\frac{1}{12\pi} F_U[C(U,V)], \qquad T_{VV} = -\frac{1}{12\pi} F_V(C(U,V))$$
$$T_{UV} = \frac{RC(U,V)}{96\pi}$$

where the operator F is defined as $F_x(y(x)) = y^{1/2}\partial_x^2(y^{-1/2})$ and R is the AdS_2 Ricci scalar. Now reverting to u, v coordinates,

$$T_{uu} = \left(\frac{\partial U}{\partial u}\right)^2 T_{UU} = T_{uu}^{(AdS)} + T_{uu}^{(m)}$$

where $T_{uu}^{(AdS)} = -\frac{1}{12\pi} F_u[\sec^2 \frac{v-u}{2}]$ is the piece due to a static mirror and $T_{uu}^{(m)}$ due to motion of the mirror,

$$T_{uu}^{(m)} = -\frac{1}{12\pi}\left(\frac{dp}{du}\right)^{1/2} \frac{d}{du}\left(\frac{dp}{du}\right)^{-1/2} + \frac{1}{12\pi}\left(\frac{dp}{du}\right)^{-1/2} \sec\left(\frac{v-u}{2}\right) \frac{d}{du}\left(\frac{\frac{d}{du}\left(\frac{dp}{du}\right)^{1/2}}{\frac{dU}{du}\sec(\frac{v-u}{2})}\right) \tag{B3}$$

$$T_{vv}^{(m)} = T_{uv}^{(m)} = 0\,. \tag{B4}$$

Related results appear in [62].

Let us model the path of the expanding bubble by a trajectory with constant acceleration α

$$\begin{aligned} x(\tau) &= \alpha^{-1}\cosh\alpha\tau \\ t(\tau) &= \alpha^{-1}\sinh\alpha\tau \end{aligned}$$

we have,

$$u(\tau) = -\alpha^{-1}e^{-\alpha\tau}, \quad v(\tau) = \alpha^{-1}e^{\alpha\tau} = -\alpha^{-2}u^{-1}(\tau) \tag{B5}$$

Now if we go to the rest frame of the bubble,

$$V(v) - U(u) = 0,$$

Then with $V(v) = v$ and $U(u) = -\alpha^2 u^{-1}$ the condition (B5) is satisfied. So for the uniformly accelerating mirror we have,

$$p(u) = -\alpha^{-2}u^{-1}.$$

The energy flux at infinity can readily be checked to be finite.

The calculation can be generalized to the SAdS black hole in $1+1$-d and the same qualitative results are obtained, though formulas are more complicated. The stress-energy tensor splits into two different components: Hawking radiation of the black hole; the contribution from the reflection from the moving mirror. The reflected radiation just alters the black hole mass as argued in the above calculation by a finite amount. We conclude the path of the bubble may be deduced from the radiation received at infinity in the AdS region.

A more realistic model would derive the boundary conditions of the fields from the detailed knowledge of the gravity theory, including the potential of figure 1. In this case the radiation reaching infinity in the AdS region will also contain information on the internal state of the bubble, and consequently, this information will show up CFT correlation functions in a bubble state.

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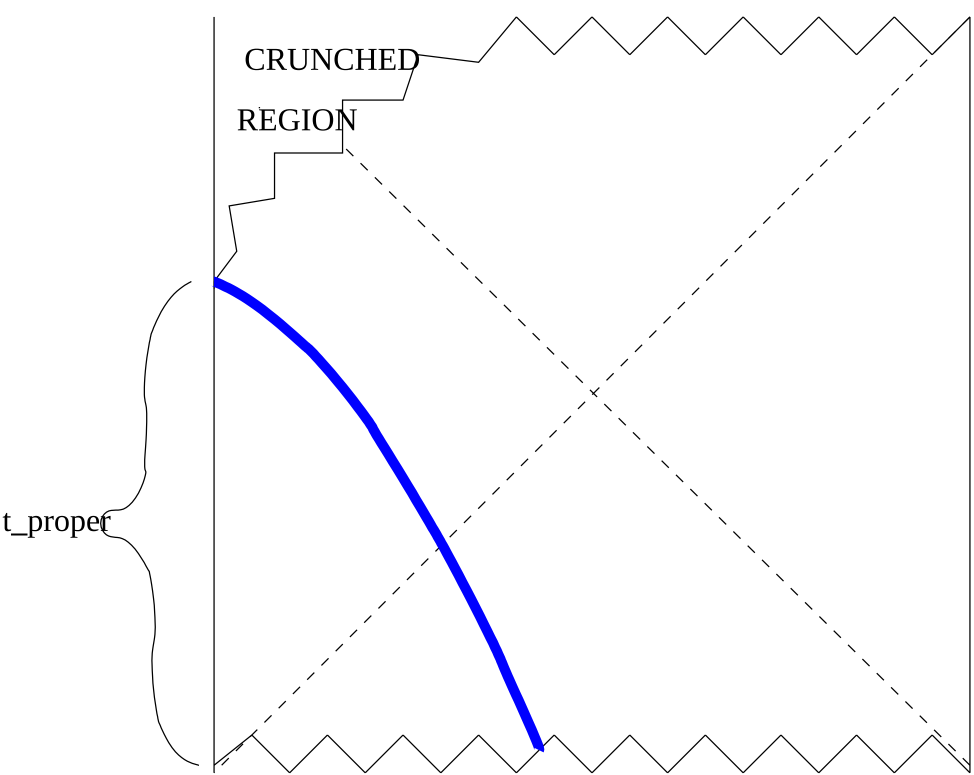
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